# 6.1 Solving Systems of Linear Equations Using Matrices

## ❖ Augmented Matrices

**Matrix (**plural: **Matrices):** a shortened way of writing a system of equations.
***M* x *N* Matrix:** a matrix has *M* rows and *N* columns.

Matrix elements are denoted: $a_{ij}$

Ex. Element $a_{23}$ is in row 2, column 3.

Ex. Write the augmented matrix for each system of linear equations.

a.) $\begin{cases} 5x - y = 1 \\ 3x + 2y = 24 \end{cases}$

b.) $\begin{cases} x - 2y + z = 10 \\ 3x + y = 5 \\ 7x + 2z = 2 \end{cases}$

Ex. Write the system of linear equations represented by the augmented matrix.

$$\left[\begin{array}{ccc|c} 7 & 0 & 4 & -13 \\ 0 & 1 & -5 & 11 \\ 2 & 7 & 0 & 6 \end{array}\right]$$

## ❖ Solving Linear Systems Using Gauss-Jordan Elimination

**Solving a system of equations:** find values of the variables that make all the equations true.

Three possible outcomes when solving a system of equations:

1.) One unique solution (Sec. 6.1)
2.) No Solution (Sec. 6.2)
3.) Infinite number of solutions (Sec. 6.2)

**Solving a System of Equations Using Gauss-Jordan Elimination:** use various "row operations" to change the augmented matrix into ***reduced row-echelon form***.

**Reduced Row-Echelon Form:**

$$\left[\begin{array}{cc|c} 1 & 0 & a \\ 0 & 1 & b \end{array}\right] \quad \text{or} \quad \left[\begin{array}{ccc|c} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \end{array}\right]$$

$x = a$
$y = b$
$(a, b)$

$x = a$
$y = b$
$z = c$
$(a, b, c)$

**The acceptable elementary row operations are:**

1.) Swap an entire row with another row: $R_i \leftrightarrow R_j$

2.) Multiply a row by a non-zero constant: $kR_i$

3.) Multiply a row by a non-zero constant and add it to another row: $kR_i + R_j$

Ex. Solve each system of equations using **Gauss-Jordan Elimination**. State the solution.

a.) $\begin{cases} -2x + 6y = -14 \\ x - 5y = 13 \end{cases}$

**Hint #1:**
You can create a **ONE** by multiplying your row by the reciprocal.

**Hint #2:**
You can create a **ZERO** by multiplying the pivot row by the opposite and adding to your row.

b.) $\begin{cases} 3y - z = -1 \\ x + 5y - z = -4 \\ -3x + 6y + 2z = 11 \end{cases}$

Ex. (#62) Sylvia invested a total of $40,000. She invested part of the money in a certificate of deposit (CD) that earns 2% simple interest per year. She invested in a stock that returns the equivalent of 8% simple interest, and she invested in a bond fund that returns 5%. She invested twice as much in the stock as she did in the CD, and earned a total of $2300 at the end of 1 year. How much principal did she put in each investment?